MODELING OF RADON DIFFUSION THROUGH SOIL

A Thesis
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Requirements for the award of the degree of

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in
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by
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Declaration:

I, the undersigned, declare that the work contained in this thesis entitled **Modelling of Radon Diffusion Through Soil**, in partial fulfillment of the requirement for the award of the degree of Master of Science, submitted in the Department of Mathematics, National Institute of Technology, Rourkela, is entirely my own work and has not previously in its entirety or part been submitted at any university for a degree, and that all the sources I have used or quoted have been indicated and appropriately acknowledged by complete references.

INDRAJIT SUARO

May 2014

This is to certify that the above statement made by the candidate is correct to the best of my knowledge.

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ABSTRACT

Radon ^{222}Rn has been identified as an important factor that could result in a health hazard. The health risks can be minimized by taking some preventive measures where the radon concentration is very high such as in some mines and homes. A study in the diffusion of the inert gas, will give us a better understanding of its possible pathways through soil into the air where the radon releases can become hazardous.

This thesis aims to investigate the Radon diffusion through soil and into air. Results obtained from two-medium model (soil and air) is compared with the one-medium model (only soil). Finite difference methos has been used here to solve the equation describing diffusion of radon through a cylindrical soil slab [1]. It is seen that the Explicit Finite Difference Method (EFDM) is effective and accurate for solving this equation, which is used to calculate the radon diffusion, which may help to assess health hazards.

Contents

1 CHAPTER 1 BACKGROUND AND MOTIVATION

1.1 Introduction

Radon is the second leading cause of lung cancer after tobacco and alcohol. Radon transport in porous materials is strongly influenced by the presence of water. Some building materials contain natural radioactivity and which can form indoor radiation exposure. According to the studies [12] [11] of the radioactive gas radon it has been noticed that, it is responsible for environmental hazards. Primarily, it originates from building materials like soil, cement, water etc.The radon concentration and radon exhalation rate have been determined by simulation in various building materials. The radon production depends on the porosity, radium concentration and the diffusion coefficient. The radon flux exhaled from this kind of materials varies from one medium to another.

It is found that, a worldwide average of 60.4 percent of total indoor radon originates from the under-ground and surrounding buildings [1]. Radon is imperceptible to the human senses, which can get into any type of building where it is able to accumulate without proper ventilation. It is reported that radon causes more deaths per year than drunk driving, earthquakes, flood, or drought. In a built environment, radon primarily originates from soil. It enters mostly through the cracks in the building materials. It is associated with many health hazards or risks, causing Lung cancer.

Many experimental researches have reported for soil radon transport [13], [14] [16], [15] . Radon transport has been modeled by [4], [16], [17], [18] and [19] using various mediums like soil, water, air etc. As we know that analytical solutions are elusive and time taking for some difficult problems. So, numerical methods have been used to obtain the solution of the present diffusion equation. Previously it was solved using finite element method (FEM) by Urosevic and Nikezic in 2003. It was also solved using finite difference method (FDM) by Dimbylow and Wilkinson in 1985 and by Savovic and Djordjevich in 2008. The advantages of one over the other method are debatable. Both are having some advantages as well as disadvantages. While FEM may be better suited for complex three-dimensional geometries,one-dimensional problems are more easily solved by using FDM [1].

In this thesis, Explicit Finite Difference Method (EFDM) is used to solve the diffusion equation of radon through a cylindrical soil slab. This was done in two different fashions,for making comparison, i.e by using a two-medium model that accounts for two distinct regions,namely soil and air,and also using a one-medium model (just soil)as proposed by Sasaki et al.(2006) [1].

1.2 Background of Radon [9]

Radioactivity was first discovered by Henri Becquerel in 1896. He has experimented it with uranium and discovered that the uranium continuously emitts radiation. He also tried to know whether this feature was only common to uranium or other elements as well. In 1897, Pierre and Marie Curie found that the elements radium, thorium and polonium shared the same effect that Henri Becquerel was experimenting. In 1898, Ernest Rutherford confirmed Becquerels findings by conducting research on radioactivity. He identified that the continuous emission of radiation are of two different types; the relatively easy absorbed α rays and the more penetrating one β rays. He also shown that these rays had a particle nature. In 1900, Friedrich Ernst Dorn discovered that radium emanates a gas. It was first called as *Niton* (the Latin word for nitens which means shining). In 1923, this gas was named as *radon*. In 1908, William Ramsay and Robert Whytlaw-Gray isolated radon. They determined its density and found that it was the heaviest known gas.

In 1902, Marie and Pierre Curie isolated the radioactive element radium. In 1903, Henri Becquerel, together with Pierre and Marie Curie won the Physics Nobel Prize.In 1911, Marie Curie discovered the elements polonium and radium, and she was awarded the Chemistry Nobel Prize.

1.3 Overview of Radon [9] , [10]

In periodic-table, Radon (Rn) is the heaviest element of the noble gas family with atomic number 86 and atomic weight of 222. It is is a naturally occurring radioactive gas, which is invisible,colorless and odorless at room temperature. It is the least abundant noble gas. It has no stable isotopes. In fact, it is found in nature in many forms. Besides ^{222}Rn; the most notable ones are ^{220}Rn that is known as thoron and ^{219}Rn, which is known as actinon. Their half-lives are so short that the only isotope that reaches the earth surface in noticeable amounts is ^{222}Rn (Al-Kazwini and Hasan, 2003). Ihe principle isotope is ^{222}Rn, since it is long-lived. The half-life period of radon is 3.825 days i.e much longer than the half-lives of ^{220}Rn and ^{219}Rn

(55.6 s and 3.96 s respectively). It is obtained from the radioactive decay of ^{226}Ra in the ^{238}U decay series by emission of an α particle.

Thoron is formed in the thorium (^{232}Th) decay series wherein ^{224}Ra decays to form it. It has a half-life of 55.6 seconds. On the other hand, Actinon is formed in the ^{235}U decay series when ^{223}Ra decays to actinon. It has a half-life of about 3.96 seconds. Radon is formed in the ground or building materials. It is having significantly more time to diffuse through the material into the indoor environment in buildings or the outdoor atmosphere. It is formed relatively close to the earths surface and it can diffuse through the soil.

All soil contains naturally occurring primordial radionuclides such as ^{238}U, ^{232}Th and ^{402}K, having long half-lives.Thus radioactivity has always been part of life. These radionuclides pose a radiological health hazard. Many researches are going on radon from its use as an environmental tracer. Radon is a gaseous descendant of radioactive ^{238}U. It undergoes a series of decay steps before it ultimately reaches its final product ^{206}Pb, which is stable. The radiations are emitted in the form of α particles, β particles, and γ rays. Many daughter products formed in the process. These emissions are the basis by which radioactivity is measured by the radiation detectors.

1.4 The sources of radon

The primary sources of radon are the rocks in the earth crust, soils on the ground, groundwater (H_2O) and natural gas. Due to climatic changes, the rocks in the earth crust break down into soil having the element uranium, due to its long half-life period. The radon naturally occurring in the environment comes from proximal distances beneath the ground. Groundwater are contained in aquifers and rocks. It is very soluble in organic solvents, such as toluene and xylene, and moderately in cold water and this property leads to higher concentrations of radon in hydrocarbon contaminants.Here is a pie chart showing the different sources of radon.

1.5 Reasons of the Study of Radon

Radon is the decay product of ^{226}Ra. By the emission of α rays it decays to ^{218}Po, which further decays to ^{214}Pb, and the process continues down a long decay chain (Fig.2.1). The elements having the properties of radon are called progeny of radon, which is also known as radon daughters. They are chemically active in nature and they are having relatively short half-lives. If humans are exposed to the radon then only the gas itself can be inhaled or exhaled. If the decay potential of radon is very high then the radon daughters resulting from the decay chain might stick into the lungs, causing different mechanism in the lung cavities or on the lungs. Radiation gets liberated when these radon daughter decay on the surface of the lung and imparts the lung dose. That increases the risk of having lung cancer. [9]

1.6 Thesis outline

The remaining parts of this thesis are as follows: Chapter 1 describes the background and overview of radon and illustrates the interaction of radiation with matter. The radioactive decay processes and its numerical calculations are discussed in Chapter 2. Chapter 4 describes the various numerical methods, out of which, some are studied here in details and some can be used as a future work on this topic.

Chapter 5 presents the measurements and modelling of radon diffusion equation by using Explicit Finite Difference Method (FEM). Comparison between two different models are also discussed in this chapter. Chapter 6 presents the numerical results, overall outcomes of the study in a summary and future works.

2 CHAPTER 2 THE RADIATION AND THE RADIOACTIVE DECAY PROCESSES

2.1 The Radioactive Decay Law

The radioactive decay laws are derived below. If there exist N of radioactive nuclei at a certain time t and a distinct decay constant, λ. The activity A of a sample of N nuclei decays proportionally to the decay constant.

$$A = \lambda N \tag{1}$$

then the decay (dN) in the sample that will disintegrate in a short time (dt) is proportional to the Total number of nuclei N present, that is:

$$dN = -\lambda N dt \tag{2}$$

where $\lambda(s^{-1})$ is the decay constant of a certain radioactive species. Eqn. (2) can be written as

$$\lambda = \frac{dN/dt}{N} \tag{3}$$

The solution of Eqn. (3) is called the exponential law of radioactive decay [10]. It is

$$N(T) = N_0 e^{-\lambda t} \tag{4}$$

where $N(t)$ and N_0 represent the radioactive nuclei present at time t and initial time respectively. One half-life is required to decay half the amount of the radioactive atoms. Here half is denoted by $T_{1/2}$. To calculate $T_{1/2}$, substitute $N = N_0/2$ to give:

$$T_{1/2} = \frac{ln2}{\lambda} \tag{5}$$

$$\Rightarrow T_{1/2} = \frac{0.6931}{\lambda} \tag{6}$$

The activity of a species is expressed in the SI unit as Bq (Becquerel), the number of disintegrations per second. It is named after Henri Becquerel. The unit that was previously used is Ci (curie), which is 3.70×10^{10} decays per second. Ci was originally the activity of 1 gram of radium [9].

2.2 Equations for the Radon Daughter Decay Chain

Radon-222 decays into charged daughter particles. We can detect this using a Geiger counter. The radioactive daughters of radon follow a decay chain. It starts with the radon isotope ^{218}Po and progressing to ^{214}Pb, ^{214}Bi and ^{214}Po, which decays into the long-lived ^{210}Pb and effectively ends the chain. The decay chain, with half-lives in parenthesis, is as follows [3]:

$$^{222}Rn \rightarrow {}^{218}Po(3.1m) \rightarrow {}^{214}Pb(26.8m) \rightarrow {}^{214}Bi(19.7m) \rightarrow {}^{214}Po(160\mu S) \rightarrow \cdots$$

Following differential equations represent the first four steps in the decay chain of Radon-222:

$$\frac{dN_1}{dt} = -\lambda_1 N_1 \tag{7}$$

$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2 \tag{8}$$

$$\frac{dN_3}{dt} = \lambda_2 N_2 - \lambda_3 N_3 \tag{9}$$

$$\frac{dN_4}{dt} = \lambda_3 N_3 - \lambda_4 N_4 \tag{10}$$

where N_1, N_2, N_3 and N_4 are the number of ^{218}Po, ^{214}Pb, ^{214}Bi and ^{214}Po atoms respectively and λ is the decay constant of each daughter. It is related to the half-life by the following equation

$$\lambda = \frac{ln2}{T_{1/2}}$$

The first equation can be solved using variable separation method. This gives the following equation:

$$N_1(t) = N_1(0)e^{-\lambda t} \tag{11}$$

where, $N_1(0)$ is the initial number of atoms before the decay is observed. Solving Eqn. (8) we get

$$N_2(t) = \frac{\int_0^t e^{\lambda_2 t}(\lambda_1 N_1(t))dt}{e^{\lambda_2 t}}$$

when Eqn. (11) is substituted in the above equation, we got the following solution:

$$N_2(t) = \frac{\lambda_1 N_1(0)}{\lambda_2 - \lambda_1}[e^{-\lambda_1 t} - e^{-\lambda_2 t}] + N_2(0)e^{-\lambda_2 t} \tag{12}$$

Note that at $t = 0$, the equation reduces to $N_2(t) = N_2(0)$. Similarly we solve the Eqn. (9) for $N_3(t)$; i.e

$$\begin{aligned}N_3(t) = \frac{\lambda_2\lambda_1 N_1(0)}{\lambda_2-\lambda_1}[\frac{e^{-\lambda_1 t}-e^{-\lambda_3 t}}{\lambda_3-\lambda_1} - \frac{e^{-\lambda_2 t}-e^{-\lambda_3 t}}{\lambda_3-\lambda_2}]+ \\ \frac{\lambda_2 N_2(0)}{\lambda_3-\lambda_2}[e^{-\lambda_2 t}-e^{-\lambda_3 t}] + N_3(0)e^{-\lambda_3 t}\end{aligned} \tag{13}$$

the solution of Eqn. (10) will be:

$$\begin{aligned}N_4(t) = \frac{\lambda_3\lambda_2\lambda_1 N_1(0)}{\lambda_2-\lambda_1}[\frac{1}{\lambda_3-\lambda_1}(\frac{e^{-\lambda_1 t}-e^{-\lambda_4 t}}{\lambda_4-\lambda_1} - \frac{e^{-\lambda_3 t}-e^{-\lambda_4 t}}{\lambda_4-\lambda_3})- \\ \frac{1}{\lambda_3-\lambda_2}(\frac{e^{-\lambda_2 t}-e^{-\lambda_4 t}}{\lambda_4-\lambda_2} - \frac{e^{-\lambda_3 t}-e^{-\lambda_4 t}}{\lambda_4-\lambda_3})]+ \\ \frac{\lambda_3\lambda_2 N_2(0)}{\lambda_3-\lambda_2}[\frac{e^{-\lambda_2 t}-e^{-\lambda_4 t}}{\lambda_4-\lambda_2} - \frac{e^{-\lambda_3 t}-e^{-\lambda_4 t}}{\lambda_4-\lambda_3}]+ \\ \frac{\lambda_3 N_3(0)}{\lambda_4-\lambda_3}[e^{-\lambda_3 t}-e^{-\lambda_4 t}] + N_4(0)e^{-\lambda_4 t}\end{aligned} \tag{14}$$

2.3 Numerical Solution of Decay Equations

Using values [3] of the half-lives of ^{218}Po, ^{214}Pb, ^{214}Bi and ^{214}Po as 185.8s, 26.83 min, 19.9 min, and 164.3 s, respctively, we get the values of the decay constant λ for the isotopes: $\lambda_1 = 3.731*10^{-3}s^{-1}$ (^{218}Po), $\lambda_2 = 4.306*10^{-4}s^{-1}$ (^{214}Pb), $\lambda_3 = 5.81*10^{-4}s^{-1}$ (^{214}Bi), and $\lambda_4 = 4.219*10^{-3}s^{-1}$ (^{214}Po). Substituting these values in Eqn. (11) to (14) we get

$$N_2(t) = -1.13N_1(0)e^{-\lambda_1 t} + (N_2(0) + 11.13N_1(0))e^{-\lambda_2 t} \tag{15}$$

$$\begin{aligned}N_3(t) = 155N_1(0)e^{-\lambda_1 t} + (3.24N_1(0) \\ +2.86N_2(0))e^{-\lambda_2 t} + (-158N_1(0) - 2.86N_2(0) + N_3(0))e^{-\lambda_3 t}\end{aligned} \tag{16}$$

Since λ_4 has a value of at least six orders of magnitude greater than the values of the other decay constants, so all the terms with the form $\frac{1}{\lambda_4-\lambda_n}$ can be approximated as $\frac{1}{\lambda_4}$ [3].

$$
\begin{aligned}
N_4(t) = \frac{\lambda_3\lambda_2\lambda_1 N_1(0)}{\lambda_2-\lambda_1}[&\frac{1}{\lambda_3-\lambda_1}(\frac{e^{-\lambda_1 t}-e^{-\lambda_4 t}}{\lambda_4}-\frac{e^{-\lambda_3 t}-e^{-\lambda_4 t}}{\lambda_4})- \\
&\frac{1}{\lambda_3-\lambda_2}(\frac{e^{-\lambda_2 t}-e^{-\lambda_4 t}}{\lambda_4}-\frac{e^{-\lambda_3 t}-e^{-\lambda_4 t}}{\lambda_4})]+ \\
&\frac{\lambda_3\lambda_2 N_2(0)}{\lambda_3-\lambda_2}[\frac{e^{-\lambda_2 t}-e^{-\lambda_4 t}}{\lambda_4}-\frac{e^{-\lambda_3 t}-e^{-\lambda_4 t}}{\lambda_4-\lambda_3}]+ \\
&\frac{\lambda_3 N_3(0)}{\lambda_4}[e^{-\lambda_3 t}-e^{-\lambda_4 t}]+N_4(0)e^{-\lambda_4 t}
\end{aligned}
\tag{17}
$$

Substituting the numerical values of the decay constants and rearranging them, we get:

$$
\begin{aligned}
N_4(t) = -2.13*10^{-8}N_1(0)e^{-\lambda_1 t} &+ (4.46*10^{-7}N_1(0)+3.94*10^{-8}N_2(0))e^{-\lambda_2 t} \\
&+(-4.67*10^{-7}N_1(0)+-3.94*10^{-8}N_2(0)+1.38*10^{-7}N_3(0))e^{-\lambda_3 t} \\
&+(-1.38*10^{-7}N_3(0)+N_4(0))e^{-\lambda_4 t}
\end{aligned}
\tag{18}
$$

In equilibrium the rates of decay of all the isotopes in the decay chain are equal. It can be represented as

$$
\lambda_1 N_1 = \lambda_2 N_2 = \lambda_3 N_3 = \lambda_4 N_4 \tag{19}
$$

Substituting all the numerical values in this equation, we get:

$$
\begin{aligned}
N_2 &= 8.66N_1 \\
N_3 &= 6.42N_1 \\
N_4 &= 8.8*10^{-1}N_1
\end{aligned}
$$

N_2 and N_3 exactly agrees well with [3] but the value of N_4 is given as $8.668*10^{-7}N_1$, which may seem to be exact matching.

3 CHAPTER 3 NUMERICAL METHODS

3.1 Introduction

Partial differential equations occur in many branches of applied mathematics, for example, in hydrodynamics, elasticity, quantum mechanics etc. The general second-order linear partial differential equation is of the form [20]

$$A\frac{\partial^2 u}{\partial x^2} + B\frac{\partial^2 u}{\partial xy} + C\frac{\partial^2 u}{\partial y^2} + D\frac{\partial u}{\partial x} + E\frac{\partial u}{\partial y} + Fu = G. \tag{20}$$

the above can be written as.

$$Au_{xx} + Bu_{xy} + Cu_{yy} + Du_x + Eu_y + Fu = G. \tag{21}$$

where A, B, C, ...,G, are all functions of x and y.
Equation of this form (1) can be classified with respect to the sign of the discriminant.

$$\bigtriangledown_s = B^2 - 4AC, \tag{22}$$

in the following way. If $\bigtriangledown_s < 0$ at a point in the (x, y) plane, the equation is said to be of *elliptic* type, to be of *hyperbolic* type when $\bigtriangledown_s = 0$ at that point , and to be of *parabolic* type when $\bigtriangledown_s > 0$.
In the following, we will discuss three simple particular cases of Eqn. (22), for the sake of completeness

$$u_{xx} + u_{yy} = 0 \tag{23}$$

$$u_{xx} - (1/c^2)u_{tt} = 0 \tag{24}$$

$$u_{xx} - u_t = 0, \tag{25}$$

where (x, y) are space coordinates and t is the time coordinate. It is easy to see that the Laplace equation is of elliptic type, that the wave equation is of hyperbolic type and that the heat equation is of parabolic type.
In a similar way, we conclude that the partial differential equation

$$xu_{xx} + u_{yy} = 0 \tag{26}$$

is
(i) parabolic if $x = 0$
(ii) elliptic if $x > 0$
(iii) hyperbolic if $x < 0$.

It is clear that the region plays an important role in the classification of partial differential equation.
In most of the cases it is easier to develop approximate solution by numerical methods. Several numerical methods have been proposed for the solution of partial differential equations, these are,
(a) Finite difference method.
(b) Method of lines.
(c) Finite element method.
(d) Finite volume method.
(e) Spectral method.
(f) Meshfree methods.
(g) Domain decomposition methods.
(h) Multigrid methods.

In the following diffusion, Finite Difference Method has been outlined.

3.2 Finite difference approximation to partial derivatives

We consider a rectangular region R in the x-y plane. We divide this region into a rectangular network side $\triangle x = h$ and $\triangle y = k$. The point of intersection of the dividing lines are called mesh points, nodal points or grid points.

$$\frac{\partial u}{\partial x} = \frac{u(x+h,y) - u(x,y)}{h} + o(h) = \frac{u(x,y) - u(x-h,y)}{h} + o(h) \tag{27}$$

$$\frac{\partial u}{\partial x} = \frac{(x+h,y) - u(x-h,y)}{2h} + o(h^2) \tag{28}$$

similarly we can define

$$\frac{\partial^2 u}{\partial x^2} = \frac{(x-h,y) - 2u(x,y) + u(x+h,y)}{h^2} + o(h^2). \tag{29}$$

If we replace $u(x,y) = u(ih, jk)$ by $u_{i,j}$, then we will get u_x and other derivatives as

$$u_x = \frac{u_{i+1,j} - u_{i,j}}{h} + o(h) = \frac{u_{i,j} - u_{i-1,j}}{h} + o(h) = \frac{u_{i+1,j} - ui - 1, j}{h} + o(h^2) \tag{30}$$

$$u_{xx} = \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2} + o(h^2) \tag{31}$$

similarly we can define

$$u_y = \frac{u_{i,j+1} - u_{i,j}}{k} + o(k) = \frac{u_{i,j} - u_{i,j-1}}{k} + o(k) = \frac{u_{i,j+1} - ui,j-1}{k} + o(k^2) \tag{32}$$

$$u_{yy} = \frac{u_{i,j-1} - 2u_{i,j} + u_{i,j+1}}{k^2} + o(k^2) \tag{33}$$

3.2.1 Elliptic Equations

Laplace's and Poisson Equations
We will develop finite difference procedure to solve Laplace's equation

$$u_{xx} + u_{yy} = 0 \tag{34}$$

in a bounded region R with boundary C. R is a rectangular region for which *u(x,y)* is known at the boundary. Assuming that an exact sub-division of R is possible, we divide this region in to a network of square mesh of side h. Replacing the derivatives in (34) by their finite approximation, [20] we have

$$\frac{1}{h^2}[u_{i-1,j} - 2u_{i,j} + u_{i+1,j}] + \frac{1}{h^2}[u_{i,j-1} - 2u_{i,j} + u_{i,j+1}] = 0$$
$$\Rightarrow u_{i,j} = \frac{1}{4}[u_{i-1,j} + u_{i+1,j} + u_{i,j+1} + u_{i,j-1}] \tag{34a}$$

This shows that the value of u at any interior mesh point is the average of its values at four neighbouring points to the left, right, above and below. The equation (34a) is known as standard 5-point formula.
Sometime we may use a formula

$$u_{i,j} = \frac{1}{4}[u_{i-1,j+1} + u_{i+1,j-1} + u_{i+1,j+1} + u_{i-1,j-1}] \tag{35}$$

which is similar to (34a). This shows that the value of $u_{i,j}$ is the average of its values at the four neighbouring diagonal mesh point and the Equation (35) is called diagonal 5-point formula. Although (35) is less accurate than (34a) but serves as reasonably good approximation for obtaining starting value at the mess points. We use the diagonal five-point formula (35) to find the initial values of u at the interior mesh point and compute $u_{3,3}$, $u_{2,4}$, $u_{4,4}$, $u_{4,2}$, and $u_{2,2}$. The value at the remaining interior points i.e. $u_{2,3}$, $u_{3,4}$, $u_{4,3}$, and $u_{3,2}$ are computed by standard 5-point formula.

Poisson Equation
Let us now consider the Poisson's equation

$$u_{xx} + u_{yy} = f(x, y) \tag{36}$$

the method of solving equation (36) is similar to Laplace equation. Here the standard 5-point formula for (36) is [20]

$$u_{i-1,j} + u_{i+1,j} + u_{i,j+1} + u_{i,j-1} - 4u_{i,j} = h^2 f(ih, jh) \tag{37}$$

using (36) at each interior mesh point, we arrive at a system of linear equation in the nodal values $u_{i,j}$ After determining the $u_{i,j}$ once, their accuracy can be improved either by using Jacobi's iterative method or using Gauss-Seidal iterative method. The processes is repeated till the two consecutive iteration become very close i.e. the difference between two consecutive iteration becomes negligibly small in order to achieve the desired level of accuracy. The iterative formula in case of Jacobi's method and Gauss-siedal method are.

$$u_{i,j}^{(n+1)} = \frac{1}{4}[u_{i-1,j}^{(n)} + u_{i+1,j}^{(n)} + u_{i,j+1}^{(n)} + u_{i,j-1}^{(n)}] \tag{38}$$

$$u_{i,j}^{(n+1)} = \frac{1}{4}[u_{i-1,j}^{(n+1)} + u_{i+1,j}^{(n)} + u_{i,j+1}^{(n+1)} + u_{i,j-1}^{(n)}]$$

As given in [20], following are the steps to be followed.
Step-1 Select the problem and draw the region according to given problem.

$$\int_a^b f(x)$$

Step-2 Then according to value of $h = 1/(nx - 1)$ divide it into meshes the crossing points are known as nodal point of the figure.

Step-3 Calculate the value of u_1, u_2, u_3,..., u_9 .

Step-4 We first use the diagonal five- point formula and compute u_5, u_7, u_9, u_1, and u_3. Thus, we obtain

$$u_5 = \frac{1}{4}(c_1 + c_5 + c_9 + c_{13}); \qquad u_7 = \frac{1}{4}(c_{15} + u_5 + c_{11} + c_{13});$$
$$u_9 = \frac{1}{4}(u_5 + c_7 + c_9 + c_{11}); \qquad u_1 = \frac{1}{4}(c_1 + c_3 + u_5 + c_{15});$$
$$u_3 = \frac{1}{4}(c_3 + c_5 + c_7 + u_5).$$

Step-5 We then compute, in the order, the remaining quantity, u_8, u_4, u_6, and u_1 by the standard five-point formula. Thus, we have

$$u_8 = \frac{1}{4}(u_5 + u_9 + c_{11} + u_7); \qquad u_4 = \frac{1}{4}(u_1 + u_5 + u_7 + c_{15});$$
$$u_6 = \frac{1}{4}(u_3 + c_7 + u_9 + u_5); \qquad u_2 = \frac{1}{4}(c_3 + u_3 + u_5 + u_1).$$

Step-6 When once all the u_i, are computed, their accuracy can be improved by using any of the iterative methods Jacobi's or Gauss-Seidal method.

3.2.2 Parabolic equation [22]

We consider the heat conduction equation.

$$C\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \tag{39}$$

C being a constant.
Let the (x, t) plane be divided into smaller rectangular by means of the sets of the lines
$x = ih$, i=0,1,2,...
$t = jk$, j=0,1,2,...
using approximations

$$\frac{\partial u}{\partial t} = \frac{u_{i,j+1} - u_{i,j}}{k}, \tag{40}$$

and

$$\frac{\partial^2 u}{\partial x^2} = \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2} \tag{41}$$

putting the equation (39) and (40) in (38) we will get

$$C\frac{u_{i,j+1} - u_{i,j}}{k} = \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}. \tag{42}$$

which can be written as

$$u_{i,j+1} = u_{i,j} + \lambda(u_{i-1,j} - 2u_{i,j} + u_{i+1,j}), \tag{43}$$

where $\lambda = k/(Ch^2)$.
This formula express the unknown function value at the (i,j+1)th interior

points in the terms of the known function values and hence it is called the *explicit formula.* It can be shown that this formula is valid only for $0 < \lambda <=$ 1/2
for $\lambda = 1/2$, equation (42) reduces to [21]

$$u_{i,j+1} = \frac{1}{2}(u_{i-1,j} + u_{i+1,j}), \tag{44}$$

which is called *Bender-schmidt recurrence relation.* In formula (41), we have used the function values along the jth row only in the approximation of $\partial^2 u/\partial x^2$. If we replace $\partial^2 u/\partial x^2$ by the average of its finite-difference approximations on jth and (j+1)th rows. Thus, [21]

$$\frac{C}{k}(u_{i,j+1} - u_{i,j}) = \frac{1}{2h^2}(u_{i-1,j} - 2u_{i,j} + u_{i+1,j} + u_{i-1,j+1} - 2u_{i,j+1} + u_{i+1,j+1}), \tag{45}$$

which gives on rearranging

$$-\lambda u_{i-1,j+1} + (2+2\lambda)u_{i,j+1} - \lambda u_{i+1,j+1} = \lambda u_{i-1,j} + (2-2\lambda)u_{i,j} + \lambda u_{i+1,j}, \tag{46}$$

where $\lambda = k/(Ch^2)$.
This formula is known as *Crank-Nicolson formula* and is valid for all the finite values of λ.
If the partial differential equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \tag{47}$$

is replaced by the finite difference equation

$$(1+r)u_{i,j+1} = u_{i,j+1} + \frac{1}{2}r(u_{i-1,j+1} + u_{i+1,j} + u_{i+1,j+1} + u_{i-1,j} - 2u_{i,j}) \tag{48}$$

where $r = k/h^2$.
In the above equation $u_{i,j+1}$, $u_{i-1,j+1}$ are unknown and all other known since they are already computed at the jth step. Hence dropping the jth and setting

$$c_{i,j} = u_{i,j} + \frac{1}{2}r(u_{i-1,j} - 2u_{i,j} + u_{i+1,j}) \tag{49}$$

equation (47) can be written as

$$u_i = \frac{r}{2(1+r)}(u_{i-1} + u_{i+1}) + \frac{c_i}{1+r} \tag{50}$$

now from equation (49) we will get

$$u_i^{(n+1)} = \frac{r}{2(1+r)}(u_{i-1}^{(n)} + u_{i+1}^{(n)}) + \frac{c_i}{1+r} \tag{51}$$

which is known as *jacobi's iteration formula* [21]. Also the formula [21]

$$u_i^{(n+1)} = \frac{r}{2(1+r)}(u_{i-1}^{(n)} + u_{i+1}^{(n)}) + \frac{c_i}{1+r} \tag{52}$$

is known as *Gauss-siedel iteration formula.*

3.2.3 Hyperbolic equation [22]

We consider the boundary value problem defined by

$$u_{tt} = c^2 u_{xx} \tag{53}$$

$$u(x,0) = f(x) \tag{54}$$

$$u_t(x,0) = \Phi(x) \tag{55}$$

$$u(0,t) = \Psi_1(t) \tag{56}$$

$$u(1,t) = \Psi_2(t) \tag{57}$$

for $0 \leq t \leq T$, which models the transverse vibration of a stretched string.
We know that

$$u_{xx} = \frac{1}{h^2}(u_{i-1,j} - 2u_{i,j} + u_{i+1,j}) + o(h^2) \tag{58}$$

$$u_{tt} = \frac{1}{k^2}(u_{i,j-1} - 2u_{i,j} + u_{i,j+1}) + o(k^2) \tag{59}$$

where
$x = ih$, i=0,1,2,..., and $t = jk$, j=0,1,2,...
Further, $u_t(x,t)$ is approximated as follows

$$u_t(x,t) = \frac{1}{2k}(u_{i,j+1} - u_{i,j-1}) + o(k^2) \tag{60}$$

putting equation (58) and (59) in equation (53), we will get,

$$\frac{1}{k^2}(u_{i,j-1} - 2u_{i,j} + u_{i,j+1}) = \frac{c^2}{k^2}(u_{i,j-1} - 2u_{i,j} + u_{i,j+1})$$

$$u_{i,j+1} = -ui, j+1 + \alpha^2(u_{i-1,j} + u_{i+1,j}) + 2(1-\alpha^2)u_{i,j} \qquad (60a)$$

formula hold's good if $\alpha < 1$, which is condition for stability.

Some known Schemes of Finite Difference Method are as follows
a)Explicit Method
b)Implicit Method

a)Explicit Method: In explicit finite difference schemes, the temperature (U) at time n+1 depends explicitly on the temperature at time n. In this method we will use forward difference for time derivative at time t_n and a second-order central difference for the space derivative at position x_j. we get the explicit finite difference discretization of equation as:

$$\frac{U_j^{n+1} - U_j^n}{\triangle\, t} = \frac{U_{j+1}^n - 2U_j^n + U_{j-1}^n}{\triangle\, x^2} \tag{61}$$

This can rearranged in the following manner:

$$U_j^{n+1} = U_j^n + \triangle\, t \frac{U_{j+1}^n - 2U_j^n + U_{j-1}^n}{\triangle\, x^2} \tag{62}$$

Since we know the values of U_{j+1}^n, U_j^n and U_{j-1}^n, so we can compute the value of U_j^{n+1}. The main advantage of explicit methods is that they are relatively simple and also computationally fast. But, the main drawback of this method is that stability is obtained only when

$$0 < \frac{\triangle\, t}{\triangle\, x^2} < \frac{1}{2} \tag{63}$$

If this condition is not satisfied, the solution becomes unstable and starts oscillating. The numerical errors are proportional to the time step and the square of the space step:

$$\triangle\, U = O(k) + O(h^2)$$

.

b)Implicit Method: In implicit finite difference schemes, the spatial derivatives $\frac{\partial^2 C(x,t)}{\partial x^2}$ are evaluated at the new time step. If we use the backward difference at time t_{n+1} and a second-order central difference for the space derivative at position x_j, we get the implicit finite difference discretization of equation as:

$$\frac{U_j^{n+1} - U_j^n}{\triangle\, t} = \frac{U_{j+1}^{n+1} - 2U_j^{n+1} + U_{j-1}^{n+1}}{\triangle\, x^2} \tag{64}$$

This can rearranged in the following manner:

$$U_j^{n+1} = U_j^n + \triangle\, t \frac{U_{j+1}^{n+1} - 2U_j^{n+1} + U_{j-1}^{n+1}}{\triangle\, x^2} \tag{65}$$

This can be rearranged so that unknown terms are on the left and known terms are on the right. We can obtain u_j^{n+1} from the other values this way:

$$u_j^{n+1} = (1 - 2r)u_j^n + ru_{j-1}^n + ru_{j+1}^n \tag{66}$$

where $r = \dfrac{\triangle\, t}{\triangle\, x^2}$ The errors are linear over the time step and quadratic over the space step: $\Delta u = O(k) + O(h^2)$.

c)Crank Nicholson Method: In this method we will use the central difference at time $t_{n+1/2}$ and a second-order central difference for the space derivative at position x_j. we get the crank Nicholson discretiation of equation as: [8]

$$\frac{u_j^{n+1} - u_j^n}{k} = \frac{1}{2}\left(\frac{u_{j+1}^{n+1} - 2u_j^{n+1} + u_{j-1}^{n+1}}{h^2} + \frac{u_{j+1}^n - 2u_j^n + u_{j-1}^n}{h^2}\right) \tag{67}$$

This formula is known as the Crank Nicholson method. We can obtain u_j^{n+1} from solving a system of linear equations:

$$(2 + 2r)u_j^{n+1} - ru_{j-1}^{n+1} - ru_{j+1}^{n+1} = (2 - 2r)u_j^n + ru_{j-1}^n + ru_{j+1}^n \tag{68}$$

The scheme is always numerically stable and convergent but usually more numerically intensive as it requires solving a system of numerical equations on each time step. The errors are quadratic over both the time step and the space step:

$$\Delta u = O(k^2) + O(h^2). \tag{69}$$

Usually the Crank Nicholson scheme is the most accurate scheme for small time steps. The explicit scheme is the least accurate and can be unstable, but is also the easiest to implement and the least numerically intensive. The implicit scheme works the best for large time steps.

4 CHAPTER 4 MODELLING OF RADON DIFFUSION EQUATION [1]

4.1 Background

As mentioned in [1] , the basic concept behind measuring the radon concentration involves observations of radon diffusion through a porous medium. The detail experiment and measurement has been given in [1] . In this thesis, we have used their model to study and understand the basics of procedure of the methodology. Two chambers having different radon concentrations are separated by this medium. In such configurations, radon diffusion occours from on chamber to another. Primarily it occours in one direction only. Ficks law describes that radon flux density J through the medium in the direction of x-axis (Sasaki et al.,2006):

$$J = -D\frac{dC(x,t)}{dx} \tag{70}$$

Where J is the Radon flux density $[Bq/m^2s]$, D is the diffusion coefficient in the medium $[m^2s^{-1}]$, where C(x,t) is the concentration of the component and $\dfrac{\partial C}{\partial x}$ is the gradient of radon concentration C $[Bqm^{-3}]$.
The differential equation for radon diffusion is

$$\frac{\partial C(x,t)}{\partial t} = D\frac{\partial^2 C(x,t)}{\partial x^2} - \lambda C(x,t) \tag{71}$$

where λ is the decay constant of radon, whose value is $2.1 \times 10^{-6}s^{-1}$. Here the radon production within the porous medium itself is neglected.

The target of the work in geometric configuration that was studied theoretically and experimentally by Sasaki etal.(2006), which is shown in Fig. 1.

The two medium diffusion model: [1]

It consists of a cylinder containing two separate mediums, namely soil aid air. Radon diffuses from one side of the cylinder with external radon concentration C_0, to the other side which is exposed to air in the detector chamber. The actual measuring system is sealed at the detector side, making it impermeable to radon, at the coordinate $x = B + L_1$. Radon starts diffusing from the soil chamber and continue diffusing through air until some equilibrium is reached. Before the diffusion start-time t=0, radon flux density through the soil slab is zero. Hence the initial condition for the diffusion equation is

$$C(x,t) = 0, t = 0; 0 < x < B + L_1 \tag{72}$$

Radon flux density increases with the increase of time. The radon flux eventually reaches a steady state distribution across the two media, for constant radon concentration at the input side. So the boundary conditions for the diffusion equation are:

$$\begin{aligned} C(x,t) &= C_0, x = 0; t > 0 \\ J &= 0, x = B + L_1; t > 0 \end{aligned} \tag{73}$$

J must be continuous across the boundary between the soil and air in the detector. Hence, at x=B, the condition $J_{soil} = J_{air}$ is presented.

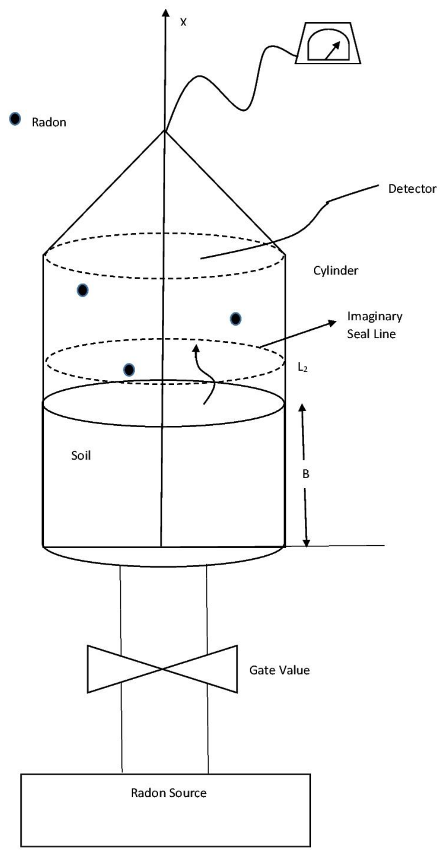

Fig. 1. Measuring system for Radon diffusion (One-medium model) [16]

The one medium diffusion model: [1]

A simplified one-medium model have been represented by Sasaki etal.(2006)from this two-medium problem. He have assumed that the diffusion coefficient for soil may be extended to air by setting an imaginary seal-line at the coordinate axis $x = B + L_2$ so that radon concentration at this line would be the same as the actual one, present at the detector end $x = B + L_1$ (Fig. 1). Hence,besides the first boundary condition at $x = 0$ in Eqn. (73), one also has to set an additional boundary condition at the imaginary seal line:

$$J = 0, x = B + L_2; t > 0 \tag{74}$$

Sasaki etal.(2006) have presented an effective length for the radon measuring device using this relation

$$\frac{D_{air}}{L_1^2} = \frac{D_{soil}}{L_2^2} \tag{75}$$

solving the above equation, we get this relation:

$$L_2 = L_1\sqrt{\frac{D_{soil}}{D_{air}}} \tag{76}$$

4.2 Numerical Methods

Here, to solve this diffusion equation, we use Explicit Finite Difference Method (EFDM) as in [1] . We use forward difference for the term $\frac{\partial C(x,t)}{\partial t}$ i.e

$$\frac{\partial C(x,t)}{\partial t} = \frac{C_{i,j+1} - C_{i,j}}{\triangle\, t}$$

and central difference for the term $\frac{\partial^2 C(x,t)}{\partial x^2}$ i.e

$$\frac{\partial^2 C(x,t)}{\partial x^2} = \frac{C_{i-1,j} - 2C_{i,j} + C_{i+1,j}}{\triangle\, x^2}$$

solving the equation using EFDM we get

$$\frac{C_{i,j+1} - C_{i,j}}{\triangle\, t} = D\frac{C_{i-1,j} - 2C_{i,j} + C_{i+1,j}}{\triangle\, x^2} - \lambda C_{i,j}$$

$$\Rightarrow C_{i,j+1} = (1 - \frac{2D\triangle t}{\triangle\, x^2} - \lambda \triangle t)C_{i,j} + \frac{D\triangle t}{\triangle\, x^2}C_{i-1,j} + \frac{2D\triangle t}{\triangle\, x^2}C_{i+1,j}$$

The two medium diffusion model:

For soil medium, the equation becomes

$$\Rightarrow C_{i,j+1} = (1 - \frac{2D_{soil}\triangle t}{\triangle\, x^2} - \lambda \triangle t)C_{i,j} + \frac{D_{soil}\triangle t}{\triangle\, x^2}C_{i-1,j} + \frac{D_{soil}\triangle t}{\triangle\, x^2}C_{i+1,j}, \quad 0 < x \leq B \tag{77}$$

For air medium, the equation becomes

$$\Rightarrow C_{i,j+1} = (1 - \frac{2D_{air}\triangle t}{\triangle\, x^2} - \lambda \triangle t)C_{i,j} + \frac{D_{air}\triangle t}{\triangle\, x^2}C_{i-1,j} + \frac{D_{air}\triangle t}{\triangle\, x^2}C_{i+1,j}, \quad B + \triangle\, x < B + L_1 \tag{78}$$

where the index i refers to the discrete positions and j refer to the discrete times, which are obtained by the step-lengths $\triangle\, x$ and $\triangle\, t$ for the coordinates x and time t, respectively. Eqn. (77) and (78) represent the formulas for $C_{i,j+1}$ at $(i, j+1)$th mesh point in terms of known values along the jth distance row. The truncation error for the equations (77) and (78) is $O(\triangle\, t, (\triangle\, x)^2)$. The truncation error can be minimized until the accuracy achieved is within the error tolerance, using a very small value of $\triangle\, x$ and $\triangle\, t$, [1]
Hence, the initial condition Eqn. (72) becomes

$$C_{i,0} = 0 \ for \ t = 0; \ 0 < x < B + L_1 \tag{79}$$

and the boundary conditions Eqn. (73) becomes

$$\begin{aligned} C_{0,j} &= C_0 \ for \ x = 0; \ t > 0 \\ C_{N,j} &= C_{N-1,j} \ for \ x = B + L_1; \ t > 0 \end{aligned} \tag{80}$$

where $N = \dfrac{(L_1 + B)}{\triangle\, x}$ is the total grid dimension of the two regions in the direction of x. The condition $J_{soil} = J_{air}$ at $x = B$ in the finite-difference form is [1]

$$\frac{D_{soil}(C_{k,j} - C_{k-1,j})}{\triangle\, x} = \frac{D_{air}(C_{k+1,j} - C_{k,j})}{\triangle\, x} \tag{81}$$

where $k = \dfrac{B}{\triangle x}$ is the grid dimension for the soil region measured in the direction of x. So, we get [1]

$$C_{k+1,j} = \frac{C_{k,j}(D_{soil} + D_{air}) - D_{soil}C_{k-1,j}}{D_{air}} \; for \; x = B + \triangle x; \; t \geq 0 \tag{82}$$

The one medium diffusion model:

In this case, the Eqn. (71) is transformed into: [1]

$$C_{i,j+1} = (1 - \frac{2D_{soil}\triangle t}{\triangle x^2} - \lambda \triangle t)C_{i,j} + \frac{D_{soil}\triangle t}{\triangle x^2}C_{i-1,j} + \frac{D_{soil}\triangle t}{\triangle x^2}C_{i+1,j}, \quad 0 < x < B + L_2 \tag{83}$$

$$C_{i,j+1} = C_{N,j}, B + L_2 < x < B + L_1 \tag{84}$$

Hence, the initial condition becomes

$$C_{i,0} = 0 \text{ for } t = 0; 0 < x < B + L_1 \tag{85}$$

and the boundary conditions are

$$\begin{aligned} C_{0,j} &= C_0 \; for \; x = 0; t > 0 \\ C_{M,j} &= C_{M-1,j} \; for \; x = B + L_2; t > 0 \end{aligned} \tag{86}$$

where $M = \dfrac{(L_2 + B)}{\triangle x}$ is the grid measured in the direction of x from $x = 0$ to imaginary sealed line with coordinate $x = L_2 + B$.

5 CHAPTER 5 RESULTS, CONCLUSION AND FUTURE WORK

Here we have assumed the data as given in [1] . Accordingly one side of the soil slab was exposed to high concentration of radon, i.e $C_0 = 40 \times 10^3 Bqm^{-3}$. And the other side was in the detector. The value $D_{soil} = 5 \times 10^{-3} cm^2/s$ was used for the radon diffusion coefficient in soil and the value $D_{air} = 0.1 cm^2/s$ was used for that in air (Sasaki et al.,2006). The length of the soil cylinder was B=5 cm and the length of the air cylinder of the detector was $L_1 = 4$ cm. The actual measuring system was sealed at the detector side at coordinate $x = L_1 + B$, which is assumed to be 9 cm (Fig. 1). Modeling the problem using Eqn. (75), we get $L_2 = 0.89cm \sim 1cm$ for the imaginary seal-line. In view of the finite difference model that has been discuss. Corresponding MATLAB programming has been developed to compute the results. Numerical results shown in Fig. 2 to 5 are the numerical results obtained by solving the diffusion equation using EFDM.
In the calculations,various step lengths have been used to achieve stability of the finite difference scheme.

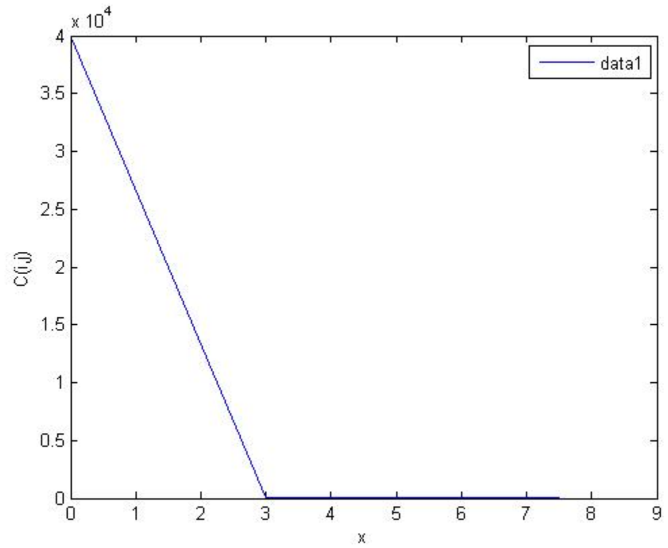

Fig. 2 $\triangle\ x = 3cm$ and $\triangle\ t = 2s$

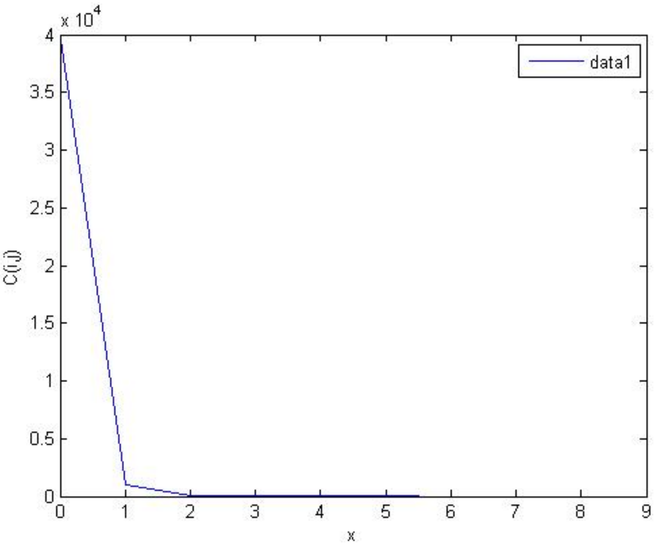

Fig. 3 $\triangle\ x = 1cm$ and $\triangle\ t = 1s$

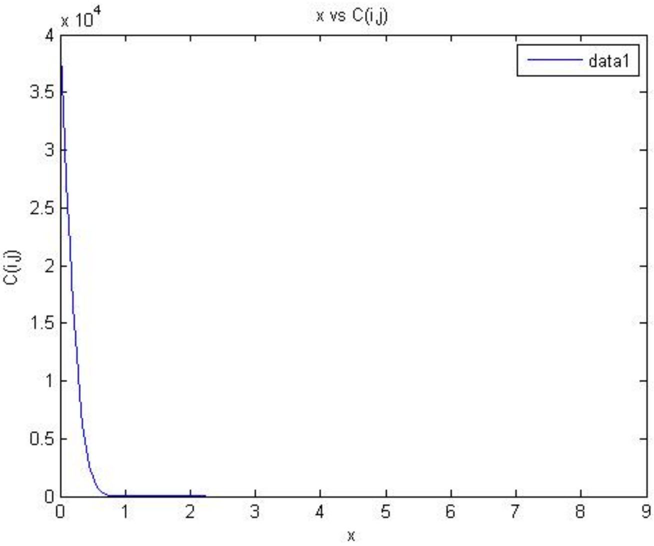

Fig. 4 $\triangle\ x = 0.05cm$ and $\triangle\ t = 0.01s$

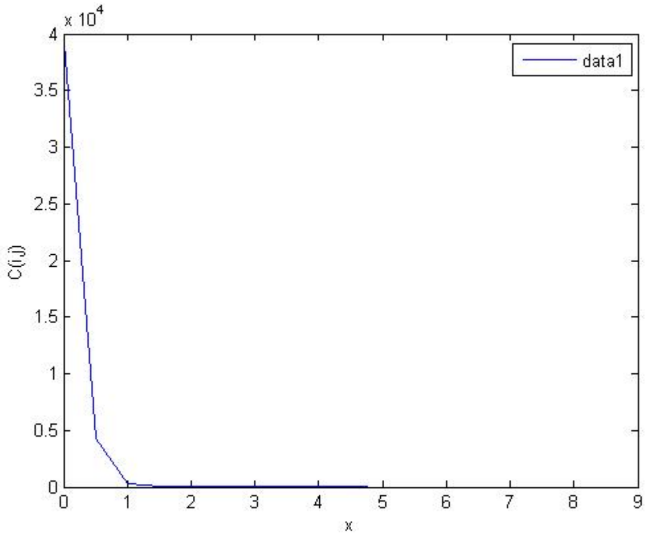

Fig. 5 $\triangle\ x = 0.05cm$ and $\triangle\ t = 0.003s$

Conclusion

Numerical solutions are computed for the diffusion equation to investigate the process of radon transport through a soil slab, following the procedure of [1]. Here one-medium diffusion model (air) and two medium diffusion model (air and soil) are used to validate the results on [1]. It was tested on various values of the step lengths $\triangle\ x$ and $\triangle\ t$. We also observed that, by reducing the values of $\triangle\ x$ and $\triangle\ t$, the curves become finer. As mentioned in [1], it is a very less rigorous approach compared to the two-medium model. In one-medium model, we do not have to know about the radon diffusion coefficient for the second region. As such we may conclude that, one medium model can give good result, which may be acceptable in practical purposes.

Future Work

It will be a challenge to model the problem as 2 or 3 dimensional, however in that case we have to use other computation efficient numerical method to handle the problem. As mentioned earlier, the medium plays a vital role in radon diffusion. So, it is also interesting to consider other type of mediums in future investigation. Further, we can also use some other numerical methods to solve it and get better results. In fact, we can change the medium of the model from air/soil/water to some other materials. For example, we can take building materials like limestone, plaster, bricks or cement can be taken to test this result.

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Nomenclature

λ	Decay constant of Radon
J	Radon flux density $[Bq/m^2s]$
D	Diffusion coefficient in the medium $[m^2s^{-1}]$
$\frac{\partial C}{\partial x}$	gradient of radon concentration C $[Bqm^{-3}]$
C_0	external radon concentration
i	Discrete position
t	Discrete time
$\triangle\ x$	step length of x
$\triangle\ t$	step length of t
M	no. of steps in one-medium diffusion model
N	no. of steps in two-medium diffusion model